

$$76) i) \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$ii) 4 \cdot \begin{pmatrix} 8 \\ 1 \end{pmatrix} + 5 \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 32 \\ 4 \end{pmatrix} + \begin{pmatrix} -5 \\ 15 \end{pmatrix} = \begin{pmatrix} 27 \\ 19 \end{pmatrix}$$

$$iii) \begin{pmatrix} 9 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -5 \end{pmatrix} = 9 \cdot 1 + 2 \cdot (-5) = 9 - 10 = -1$$

$$77) i) 5 \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \\ -10 \end{pmatrix}$$

$$ii) 3 \cdot \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix} - 5 \cdot \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 15 \\ -3 \\ 9 \end{pmatrix} - \begin{pmatrix} 10 \\ -10 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 4 \end{pmatrix}$$

$$iii) \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ -7 \end{pmatrix} = 9 + 1 - 14 = -4$$

$$78) i) (-3) \cdot \begin{pmatrix} x+11 \\ y \\ z-1 \end{pmatrix} = \begin{pmatrix} -3(x+11) \\ -3y \\ -3(z-1) \end{pmatrix} = \begin{pmatrix} -3x-33 \\ -3y \\ -3z+3 \end{pmatrix}$$

$$ii) \frac{x}{3} \cdot \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix} - \frac{2x}{3} \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{5x}{3} \\ -\frac{x}{3} \\ x \end{pmatrix} - \begin{pmatrix} \frac{2x}{3} \\ -\frac{4x}{3} \\ 2x \end{pmatrix} = \begin{pmatrix} x \\ x \\ -x \end{pmatrix}$$

$$78) \text{ iii) } \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3x - y + 2z$$

$$79) \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right) \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$80) \text{ i) } \begin{pmatrix} 0 & 0 & 0 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \cdot 3 - 0 \cdot 1 + 0 \cdot 5 \\ -2 \cdot 3 - 2 \cdot 1 + 2 \cdot 5 \\ 0 \cdot 3 - 0 \cdot 1 + 0 \cdot 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 14 \\ 0 \end{pmatrix}$$

$$\text{ii) } \begin{pmatrix} 0 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \cdot 3 - 2 \cdot 1 + 0 \cdot 5 \\ 0 \cdot 3 - 2 \cdot 1 + 0 \cdot 5 \\ 0 \cdot 3 - 2 \cdot 1 + 0 \cdot 5 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$$

$$\text{iii) } \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \cdot 3 - 0 \cdot 1 + 0 \cdot 5 \\ 0 \cdot 3 - 2 \cdot 1 + 0 \cdot 5 \\ 0 \cdot 3 - 0 \cdot 1 + 2 \cdot 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 10 \end{pmatrix}$$

$$81) \begin{pmatrix} 1 & 2 & 3 \\ 1 & -7 & 5 \\ 6 & 0 & -4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 2y + 3z \\ x - 7y + 5z \\ 6x - 4z \end{pmatrix}$$

$$82) \quad i) \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1x + 2y + 3z \\ 0x + 0y + 0z \\ 0x + 0y + 0z \end{pmatrix} = \begin{pmatrix} x + 2y + 3z \\ 0 \\ 0 \end{pmatrix}$$

$$ii) \begin{pmatrix} 0 & 0 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & -4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0x + 0y + 3z \\ 0x + 0y + 5z \\ 0x + 0y - 4z \end{pmatrix} = \begin{pmatrix} 3z \\ 5z \\ -4z \end{pmatrix}$$

$$iii) \begin{pmatrix} 1 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 0y + 0z \\ 0x - 7y + 0z \\ 0x + 0y - 4z \end{pmatrix} = \begin{pmatrix} x \\ -7y \\ -4z \end{pmatrix}$$

$$83) \quad i) \begin{pmatrix} 1 \\ -7 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x - 7y + 5z$$

$$ii) \begin{pmatrix} 1 & -7 & 5 \\ 1 & -7 & 5 \\ 1 & -7 & 5 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x - 7y + 5z \\ x - 7y + 5z \\ x - 7y + 5z \end{pmatrix}$$

$$iii) \begin{pmatrix} 1 & 1 & 1 \\ -7 & -7 & -7 \\ 5 & 5 & 5 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + y + z \\ -7x - 7y - 7z \\ 5x + 5y + 5z \end{pmatrix}$$

$$84) \quad A = \begin{pmatrix} 1 & -2 & -1 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 3 & -5 \\ -2 & -4 & 7 \end{pmatrix}$$

$$A^{-1} \cdot A = I$$

$$A \cdot A^{-1} = I$$

$$\begin{array}{c|ccc}
 & 1 & -2 & -1 \\
 A & 3 & 1 & 2 \\
 A^{-1} & 2 & 0 & 1 \\
 \hline
 1 & 2 & -3 & 1 & 0 & 0 \\
 1 & 3 & -5 & 0 & 1 & 0 \\
 -2 & -4 & 7 & 0 & 0 & 1
 \end{array} = I$$

$$85) \quad A \cdot \vec{x} = \vec{b} \quad \Rightarrow \quad \vec{x} = A^{-1} \cdot \vec{b} \quad \vec{b} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -3 \\ 1 & 3 & -5 \\ -2 & -4 & 7 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 2-6+3 \\ 2-9+5 \\ -4+12-7 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} = \vec{x}$$

$$86) \quad A \cdot \vec{x} = \vec{b}$$

$$A = \begin{pmatrix} 1 & -2 & -1 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix} \quad \vec{x} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

$$A \cdot \vec{x} = \begin{pmatrix} 1 & -2 & -1 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1+4-1 \\ -3-2+2 \\ -2-0+1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \vec{b}$$

87)

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ \hline 1 & -\frac{1}{7} & \frac{1}{3} & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{7} & 0 & 0 & -\frac{1}{7} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{7} & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 1 \end{array}$$

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{7} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

88)

$$x + y + z = 6$$

$$2x - 2y + 3z = 7$$

$$3x - 4y + 2z = 1$$

$$A \cdot \vec{x} = b$$

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 3 \\ 3 & -4 & 2 \end{pmatrix}$$

$$b = \begin{pmatrix} 6 \\ 7 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 3 \\ 3 & -4 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1+2+3 \\ 2-4+9 \\ 3-8+6 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \\ 1 \end{pmatrix}$$

$$89) \quad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{array}{c|ccc} & 1 & 0 & 0 \\ & 0 & 0 & 1 \\ & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{array}$$

$$\begin{array}{c|ccc} & 1 & 0 & 0 \\ & 0 & 0 & 1 \\ & 0 & 1 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{c|ccc} & 1 & 0 & 0 \\ & 0 & 0 & 1 \\ & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}$$

↓
 A^{-1}

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = A$$

$$90) \text{ i) } \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+z \\ x+y+z \\ x+y+z \end{pmatrix}$$

$$\text{ii) } \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \cdot x + 1 \cdot y + 0 \cdot z \\ 0 \cdot x + 0 \cdot y + 1 \cdot z \\ 1 \cdot x + 0 \cdot y + 0 \cdot z \end{pmatrix} = \begin{pmatrix} y \\ z \\ x \end{pmatrix}$$

$$\text{iii) } \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x - y \\ y - z \\ -x + z \end{pmatrix}$$